Material removal and surface generation in longitudinaltorsional ultrasonic assisted milling

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Abstract: Ultrasonic Machining (UM) is extensively used in processing of difficult 1 2 to cut materials due to its superior performance. However, the mechanics of this process is still not fully understood when superimposed on other machining operations. In this 3 paper, Longitudinal-Torsional Ultrasonic Assisted Milling (LTUAM) is introduced for 4 machining of a high strength titanium alloy Ti-6Al-4V. The separation geometrical 5 characteristics between the tool and workpiece are studied analytically. Moreover, the 6 7 texturing generation mechanism of LTUAM is firstly analyzed through a theoretical 8 model. The proposed method considers the influence of 3D tool topography, which can accurately simulate the ultrasonic surface topography. Related experiments of the 9 10 generated cutting force and the surface topography were conducted to investigate the machining characteristics. The results showed that compared with Conventional 11 12 Milling (CM), a noticeable decrease of the cutting force was observed in LTUAM. This can be explained from the perspective of tool workpiece contact rate model. Micro 13 dimpled surface textures were successfully fabricated on Ti-Al6-4V using LTUAM 14 technique. The surface test results indicated that the surface micro-hardness was 15 enhanced between 6.34% and 13.22% compared with CM. This research provides 16 guidance for the application of ultrasonic machining of textured surfaces. 17

18 **Keywords:** Longitudinal-torsional ultrasonic assisted milling; Material removal 19 process; Cutting force; Micro dimpled textures; Surface micro-hardness.

Nomencla	ture		
СМ	conventional milling	UM	ultrasonic machining
LTUAM	longitudinal-torsional ultrasonic assisted mit	lling	
R	tool radius(mm)	$v_{ m f}$	feed velocity (mm/min)
β	tool helix angle (°)	α	clearance angle of the tool (rad)
κ	angle of the secondary cutting edge of the tool (rad)	fz	feed per tooth(mm/Z)
ts	separation time at each ultrasonic vibration cycle (s)	r	tool-workpiece contact rate
W _r	tool rotation angular velocity (rad/s)	Т	ultrasonic vibration period

			(S)								
<mark>N</mark>	number of tool teeth	<mark>h</mark>	height of the ridge(µm)								
$X_{\rm w}Y_{\rm w}Z_{\rm w}$	workpiece coordinate system	$X_{\rm f}Y_{\rm f}Z_{\rm f}$	feed coordinate system								
			coordinate system								
$X_{\rm t}Y_{\rm t}Z_{\rm t}$	tool coordinate system	expanded along the cutting									
			direction								
$t_{\rm A}, t_{\rm D}$	start and end moments at each ultrasonic cycle (s)										
$\theta_{\mathrm{m,n}}, r_{\mathrm{m,n}}$	the radian and radius at position (m, n)										
$(\mathbf{x}_{0}, \mathbf{y}_{0}, \mathbf{z}_{0})$	initial position $O_{\rm f}$ of feed coordinate system with point $O_{\rm w}$ of workpiece										
(x_0, y_0, z_0)	coordinate system										
$(x_w(t), y_w(t))$	tool tip kinematic trajectories in $X_w Y_w Z_w$										
$(x_t(t), y_t(t))$	tool tip kinematic trajectories in $X_t Y_t Z_t$										
$(x_t(t), z_t(t))$	tool tip kinematic trajectories in $O_1 X_1 Z_1$										
$(x_{m,n}, y_{m,n},$	discrete coordinate points of the tool cutting edge in $X_t Y_t Z_t$										
l _{m,n}	arc length that the tool is expanded in the circumferential direction										
$ heta_{ m m,n},r_{ m m,n}$	radian and radius at position (<i>m,n</i>)										
А,В	amplitudes of longitudinal and torsional ultrasonic vibration (μ m)										
$\varphi_1, \; \varphi_2$	initial phase of longitudinal and torsional vibration (rad)										
w _f	ultrasonic vibration angular velocity (rad/s)										
$V_{\rm t}, V_{\rm h}$	cutting velocity along the cutting edge and perpendicular to the cutting edge (mm/s)										
$v_{\rm a}, v_{\rm r}$	ultrasonic velocity of longitudinal and torsional vibration (mm/s)										
$v_{\rm c}$	cutting velocity of the tool (mm/s)										
λ_1	ratio of the ultrasonic frequency to the spindle rotation velocity										
λ_2	ratio of the ultrasonic frequency to the tool tooth frequency										
d	distance between two adjacent dimples alon	g the cutti	ng velocity (µm)								
δ_1	phase shift distance of the adjacent dimples	between t	he single tooth(rad)								
δ_2	phase shift distance of the adjacent dimples between the adjacent tooth(rad)										
<mark>η</mark>	maximum opening angle between the tool p	ath and th	e horizontal plane(rad)								
d_1, d_2	distance between the highest point of the ridge and the lowest point of adjacent										
w1) w2	micro dimple(µm)										
F ^w _{of}	translational vector from $X_w Y_w Z_w$ to $X_f Y_f Z_f$										
R_{w}^{t}	transformation matrix from $X_w Y_w Z_w$ to $X_t Y_t Z_t$										
$\pmb{R}_{\mathrm{f}}^{\mathrm{t}}$	transformation matrix from $X_f Y_f Z_f$ to $X_t Y_t Z_t$										

20 **1. Introduction**

21 Titanium alloys including Ti-Al6-4V are now being widely used in different industries 22 such as aviation, aerospace, optics and biomedical engineering due to their excellent mechanical properties. However, some studies demonstrate that the inherent 23 24 characteristics of Ti-Al6-4V material such as the low thermal conductivity and high 25 chemical reactivity result in accelerated tool wear and poor surface quality, e.g. see [1]. In order to address this problem, some non-traditional methods are applied to machine 26 27 difficult to cut materials. Typical methods include electron beam machining, electrical 28 discharge machining, laser machining, ultrasonic machining and others as explained in 29 [2-4]. Among them, ultrasonic machining is significantly efficient in machining with 30 advanced, efficient, low machining cost and environmentally benign characteristics.

31 Ultrasonic Machining (UM) refers to a micron-level vibration assisted machining 32 method where an additional excitation vibration (about 20 kHz) is generated from a tool or a workpiece, e.g. see [5-7]. It combines traditional machining such as turning [8,9], 33 34 milling [10,11], grinding [12,13], drilling [14,15], polishing [16] with high-frequency ultrasonic vibration, which belongs to a composite machining method as studied in 35 36 [13,17,18]. Besides, this technology has significant advantages in reducing cutting force, improving surface quality, prolonging tool life and eliminating burrs as described 37 in [19,20]. At present, ultrasonic vibration assisted machining is widely used in the 38 processing of difficult to cut materials, such as optical glass [21], nickel alloy [22], 39 titanium alloy [23-25] and composite materials [12,26,27]. 40

When compared with conventional machining, the most essential feature of UM is that 41 it has a high-frequency relative vibration, which allows the tool and the workpiece to 42 43 periodically separate during machining. This vibration transforms the traditional continuous cutting into high-frequency intermittent cutting. Moriwaki et al. [8] first 44 applied one dimensional ultrasonic vibration cutting to turning operation. Nath et al. 45 46 [28] proposed a contact rate model in ultrasonic assisted turning. In this model, the influence of machining and vibration parameters on the contact rate were studied. 47 48 Experimental results showed that compared with conventional turning, tool life was 49 increased by 4 to 8 times and surface finish quality was also significantly improved. 50 Brehl et al. [29] investigated basic kinematic relationships of linear and elliptical ultrasonic vibration assisted cutting. The results indicated that a reduction of cutting 51 52 forces and chip thickness was due to a periodic separation between the tool rake face and workpiece. In the ultrasonic vibration assisted turning, the separation mechanism 53 54 between tool and workpiece is relatively simple. And this technique is widely used in 55 one-dimensional and two-dimensional ultrasonic turning as described in [20].

56 However due to the continuous variation of cutting thickness and tool cutting velocity, 57 an introduction of ultrasonic vibration makes the separation mechanism of milling complex. Feng et al. [30] analyzed the tool-workpiece separation and proposed three 58 typical separation mechanisms. Through these analyses, Feng *et al.* [31,32] proposed 59 novel models for the ultrasonic assisted milling to predict temperature and residual 60 stress fields. Experimental results showed that high ultrasonic amplitudes and spindle 61 speeds were the primary parameters that causing compressive residual stresses. In order 62 to reveal the reason of cutting force reduction in the feed direction ultrasonic milling, 63 64 Ni et al. [24] developed a kinematic model to describe tool and workpiece motions. The cutting time model was established through the kinematic analysis of the milling cutter 65 66 and workpiece. The milling experiments showed that the ultrasonic force has a discrete 67 pulse nature and when compared with CM, the amplitude of ultrasonic cutting force

68 and surface roughness were lower.

69 Mathematical modelling and simulation serve as a theoretical support for experimental investigations as convincingly argued in [33-35]. The surface finish generated by CM 70 is characterised by the tool marks due to the trochoidal trajectories of the tool tip whilst 7172 the ultrasonic surface finish is complex owing to the high-frequency vibration from the 73 tool or the workpiece. Guo et al. [36] invented a tertiary motion generator for the elliptical vibration assisted turning. Subsequently, Guo et al. [17] investigated the 74 influence of tool material on the elliptical vibration textures. It was noted that compared 75 with the carbide inserts, better defined surface features can be achieved with the 76 diamond insert due to a lesser elastic deformation and ploughing. Wang et al. [37,38] 77 evaluated wear conditions of a grinding wheel using the fractal analysis approach. The 78 experiments indicated that the fractal dimension of grinding wheel topography is highly 79 80 related to the grain fracture, grain pullout, and wheel loading. Zhou et al. [39] developed a 3D ground surface topography generation method for the ultrasonic 81 assisted grinding. Through those models, typical features of the ultrasonic ground 82 surfaces analyzed. In the high-speed ultrasonic vibration cutting, Peng et al. [40] 83 established a theorical model to analyze the intermittent cutting mode. Experimental 84 85 results demonstrated that the high-speed ultrasonic vibration cutting has a better performance in reducing the cutting force, improving the tool life and surface finish 86 87 quality when compared with CM. Zhang et al. [41] investigated the surface integrity using rotary ultrasonic elliptical end milling. It was found that the subsurface deformed 88 layer and surface micro-hardness increased with the improved amplitude at low cutting 89 speeds. According to the studies of Chen *et al.* [42], the ultrasonic surface textures were 90 significantly affected by machining and ultrasonic vibration parameters. In addition, 91 92 Shen et al. [43], Zhu et al. [44] also found that the micro textures can improve the surface tribological properties, especially the surface load capacity. 93

94 Recently, some researches have demonstrated that ultrasonic surface topographies are affected by tool-workpiece contact processes. Zhang et al. [45] generated the structured 95 surfaces using two-stage vibration assisted turning. The experimental results indicated 96 that the structure density defined by the aspect ratio of the micro dimples were 97 approximated well by the simulation. Xu et al. [46] proposed a new rotary ultrasonic 98 99 texturing technique and hybrid periodic micro/nano-textures were successfully 100 fabricated on machined surfaces with one-point diamond tool. Kurniawan et al. [47] developed a two-frequency elliptical vibration texturing method, which improved the 101 surface finish quality compared to conventional texturing method. Furthermore, 102 103 Kurniawan et al. [48] and Lotfi et al. [49] investigated the structured patterns during the fabrication of microgrooves using a three-dimensional elliptical vibration (3D-EVT) 104 method. Lu et al. [50] analyzed the effects of interference on the surface texture 105 generation in the ultrasonic end milling. It was found that the feed interference greatly 106 improved the surface texture so that the connectivity of the texture surface was 107

108 enhanced. Yang et al. [19,51] developed abrasive workpiece contact rate model in tangential ultrasonic vibration assisted CBN grinding of ZrO₂ ceramics and proposed 109 an ultrasonic surface topography model. In addition, a contrast experiment was 110 undertaken to investigate the surface morphology of ultrasonic grinding and common 111 grinding. In addition, Yan et al. [52] applied an external sinusoid excitation on the wheel 112 113 to mitigate regenerative chatter in plunge grinding. In order to obtain a better surface quality, Ni et al. [1,23] applied minimum quantity lubrication(MQL) technique to 114 ultrasonic milling. It was observed that the surface quality in UM&MQL was enhanced 115 by about 20-30%. Feng et al. [53,54] investigated the conditions of surface roughness 116 and tool wear. It was found that a lower spindle speed or a higher vibration amplitude 117 will reduce the surface roughness. While the tool wear was accelerated at a high 118 ultrasonic frequency and cutting speed. 119 120 Up to now, a comprehensive study on material remove process and surface generation

Up to now, a comprehensive study on material remove process and surface generation mechanism of LTUAM is not available in the literature. Moreover, the effects of processing parameters, and particular, ultrasonic parameters on surface topography are ambiguous. In this investigation, the machining characteristics of material remove rate and the micro dimple generation process in LTUAM are analyzed through simulations and experiments. Specifically, tool angles including the clearance angle and the secondary cutting edge of the cutter are considered.

127 The remainder of the paper is organized as follows. In Section 2, the kinematics of material removal process of LTUAM is modelled and analyzed with respect to the tool 128 129 tip trajectories. Then an analytical model for the tool-workpiece contact time is established. Furthermore, the micro dimpled surface generation process is investigated 130 131 through numerical simulation in Section 3. Also, the effects of process parameters and tool angle on ultrasonic surface topographies are investigated. In Section 4 the 132 experimental setup and procedure are introduced. In Section 5, some experimental 133 134 investigations including cutting force, surface topographies and surface micro-hardness are presented, which is a good validation of the theoretical model. Some conclusions 135 are drawn in Section 6. 136

137 2. Kinematic analysis of LTUAM

In order to understand material removal mechanism in the LTUAM, first the kinematic modelling and analysis are undertaken, specifically trajectories of the tool tip are modelled and analyzed. Then, the material removal process in each ultrasonic cycle of LTUAM is discussed through the tool workpiece contact rate model.

142 2.1 Analysis of the tool tip trajectories in LTUAM

143 The workpiece coordinate system $(X_w Y_w Z_w)$ is fixed on the workpiece as shown in Fig. 144 1(b). The feed coordinate system $(X_f Y_f Z_f)$ moves along the feed velocity with the

- 145 workpiece coordinate system. The tool coordinate system $(X_tY_tZ_t)$ rotates around Z_f
- 146 axis of the coordinate system $X_f Y_f Z_f$. In CM, the motion of the milling cutter with the 147 workpiece consists of the feed motion and the tool rotation. The tool tip kinematic
- 148 trajectories in CM can be expressed as:

149
$$\begin{cases} x_{w}(t) = x_{0} + v_{f}t + R\sin(w_{r}t), \\ y_{w}(t) = y_{0} + R\cos(w_{r}t), \\ z_{w}(t) = z_{0}, \end{cases}$$
(1)

where (x_0, y_0, z_0) is the initial position O_f of feed coordinate system with the point O_w of workpiece coordinate system, v_f is the feed velocity (mm/min), R is the tool radius (mm), w_r is the tool rotation angular velocity (rad/s), t denotes the motion time of the tool(s).

154 LTUAM adds longitudinal and torsional vibration to the directions of the tool axis and 155 the cutting velocity, respectively, which is the main feature of two-dimensional 156 ultrasonic vibration assisted machining. The tool tip kinematic trajectories of LTUAM 157 in $X_t Y_t Z_t$ can be expressed as:

158
$$\begin{cases} x_{t}(t) = R \sin(w_{r}t + A \sin(w_{f}t + \varphi_{1})/R), \\ y_{t}(t) = R \cos(w_{r}t + A \sin(w_{f}t + \varphi_{1})/R), \\ z_{t}(t) = B \sin(w_{f}t + \varphi_{2}), \end{cases}$$
(2)

where *A* and *B* are the amplitudes of torsional and longitudinal ultrasonic vibration (μ m), φ_1 and φ_2 are the initial phases of longitudinal and torsional vibration (rad), *w*_f represents the ultrasonic vibration angular velocity (rad/s).

162 According to Eqs (1) and (2), the tool tip kinematic trajectories of LTUAM in $X_w Y_w Z_w$ 163 can be calculated by:

164
$$\begin{cases} x_{w}(t) = x_{0} + v_{f}t + R\sin(w_{r}t + A\sin(w_{f}t + \varphi_{1})/R), \\ y_{w}(t) = y_{0} + R\cos(w_{r}t + A\sin(w_{f}t + \varphi_{1})/R), \\ z_{w}(t) = z_{0} + B\sin(w_{f}t + \varphi_{2}). \end{cases}$$
(3)

As shown in Figs 1(c) and (d), when A=0 and B=0, the longitudinal and torsional 165 vibration disappear, and machining method becomes CM. When A=0, $B=4\mu m$, 166 167 machining method belongs to LUAM, which is more common in drilling as discussed in [55]. While under the conditions of $A=4\mu m$, $B=4\mu m$, tool tip trajectories become 168 more complex. In each ultrasonic cycle, the radial motion of the milling cutter can be 169 regarded as a linear movement owing to the reason that the cutting velocity and 170 ultrasonic vibration velocity are much higher than the feed velocity. In order to facilitate 171 the analysis, the influence of the feed velocity is neglected here. A local coordinate 172system $O_1 X_1 Z_1$ is set to monitor the tool tip tracks along the axis of the milling cutter. 173 O_1 is the equilibrium position at the initial moment of the cutting edge in ultrasonic 174

vibration cycle. Z_1 axis is parallel to the tool axis, and X_1 axis is perpendicular to Z_t 175axis and tangent to the tool radius. Therefore, the tool tip kinematic trajectories in 176

177 $O_1 X_1 Z_1$ can be expressed as:

$$\begin{cases} x_1(t) = v_c t + A \sin(w_f t + \varphi_1), \\ y_1(t) = B \sin(w_f t + \varphi_2). \end{cases}$$
(4)

178179



Tool tip trajectorles in LUAM --- Tool tip trajectories in CM



180

182Figure 1. Modelling of the tool tip trajectories; (a) schematic of the LTUAM process; (b) 183 establishment of the coordinate system; (c) and (d) are the tool tip trajectories in LUAM and LTUAM respectively for f = 20kHz, and n = 2000rpm. 184

In stable machining process, the rotary motion of the tool can be regarded as a uniform circular motion. And the cutting velocity v_c is constant and can be evaluated from a simple formula,

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$$v_{\rm c} = \frac{2\pi nR}{60}.$$
 (5)

In $O_1X_1Z_1$, the trajectories of partially enlarged figure in LUAM and LTUAM are shown in Figs 1(c) and (d), respectively. From the perspective of kinematics, the tool tip trajectories in CM are straight lines. The tool tip trajectories in the longitudinal vibration milling are sine curves, while the tool tip trajectories in LTUAM are inclined sinusoids. As shown in Fig. 2, GH and G_sH_s are the tool cutting edge boundaries in LTUAM. The velocities v_r and v_a induced by ultrasonic vibration can be calculated as:



Figure 2. Kinematic analysis of LUTAM with schematic diagrams of the milling process for (a) CM
 and (c) LTUAM; velocity decomposition in the directions along the cutter and perpendicular to the
 cutter for (b) CM and (d) LTUAM.

- As shown in Figs 2(a) and (b), in CM, the tool and workpiece kinematics include the
- 202 feed motion and spindle rotation. The cutting speed is decomposed in the directions
- along the cutter and perpendicular to the cutter and can be expressed as:

204
$$\begin{cases} V_{\rm t} = v_{\rm c} \cos\beta, \\ V_{\rm h} = v_{\rm c} \sin\beta, \end{cases}$$
(7)

where V_t and V_h are the cutting velocity along the cutting edge and perpendicular to the cutting edge (m/s). In LTUAM, apart from the feed velocity and cutting speed, the motions of the tool with the workpiece are the torsional and longitudinal vibration velocities. As shown in Figs 2(c) and (d), the cutting speed and ultrasonic vibration velocities are decomposed and can be written as:

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$$\begin{cases} V_{\rm t} = (v_{\rm c} + v_{\rm r})\cos\beta + v_{\rm a}\sin\beta, \\ V_{\rm h} = (v_{\rm c} + v_{\rm r})\sin\beta - v_{\rm a}\cos\beta, \end{cases}$$
(8)

where v_r and v_a are respectively the velocities caused by torsional and longitudinal ultrasonic vibration (m/s).

213 **2.2 Analysis of the material removal process in LTUAM**

In machining, chips are generated when the cutting edge of the milling cutter and the 214 215workpiece move in the direction along the tool helix angle as seen in [56]. In Eq. (7), $v_{\rm c}$ and β are constant values in CM. As shown in Fig. 3(a), it is assumed that the tool 216 217and workpiece are at position A in the initial moment where the tool and the workpiece are in contact. As time goes by, positive V_t means the tool is chasing the cutting face. 218 Hence the cutting edge of the milling cutter reaches position B and chips are generated. 219 220 In contrast, during LTUAM, the tool cutting edge will periodically change between GH 221 and G_sH_s due to the effect of ultrasonic vibration, thereby the milling cutter and the 222 workpiece will continuously contact and separate. Specifically, as pointed by Feng *et* 223 al. [31], the conditions of contact or separation between the tool and the workpiece 224 should be determined by the combination of velocity and displacement of the tool with 225 the workpiece. Therefore, in order to determine whether the tool contact with the 226 workpiece, as shown in Fig. 3(b), it is assumed that the tool and the workpiece just 227 contact with each other at position A and $t_A = kT$, (k = 0, 1, 2, ...), where T stands for 228 ultrasonic vibration period. And B and C are the positions where V_t is zero. Based on 229 the Fig. 3(c), the value of V_t of LTUAM is positive from position A to position B, 230 which means that the tool is chasing the cutting face. In this time, the tool contacts with the workpiece. However, from position B to position C, the value of V_t in LTUAM is 231 232 negative (see Fig 3. (c)). And the tool comes away from the workpiece. Hence the tool 233 is separated with the workpiece. Finally, the tool returns to the equilibrium position D from the farthest position C and one ultrasonic cycle is completed. The tool moves 234 235 backward and forward continuously with the workpiece considering the ultrasonic vibration, which explains the kinematics of LTUAM. 236

As shown in Fig. 3(d), in order to determine whether there is no cutting occurrence at a specific helix angle, first the intersection of the milling cutter trajectory with the X_1 - axis should be determined. The cutting edge moves from point A to point D in an ultrasonic cycle. Since points A and D are on the X_1 -axis, Eq. (9) can be formulated as:

241
$$\begin{cases} B \sin(w_{\rm f} t_{\rm A} + \varphi_2) = 0, \\ t_{\rm D} = t_{\rm A} + T, \end{cases}$$
(9)

where t_A and t_D are start and end moments at each ultrasonic cycle (s), *T* denotes as the ultrasonic vibration period (s).

When the cutting edge moves to point B at time $t_{\rm B}$, the cutting edge of the tool and tool tip motion trajectories are tangent. At time $t_{\rm B}$, the combined velocity of the tool in the direction perpendicular to the cutting edge is zero, which can be given by:

 $(11 \pm Aw \cos(w) t_{-} \pm a)$







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Figure 3. Kinematics of LTUAM process; (a) relative motion of CM between tool and workpiece; (b) relative motion of LTUAM between tool and workpiece; (c) critical cutting velocity with f = 20 kHz, n = 1000rpm; (d) intermittent cutting process in adjacent vibration cycles in LTUAM; (e) material removal process in each vibration cycle.

Using Eqs (4) and (10), the coordinate of point B can be determined finally. Then the cutting edge moves from point B to point E, and its motion path is always on the left side of the line BE. Thence the cutting edge and the workpiece are always in a separate state. From Eq. (9), the coordinate of point D can be obtained as the intersection of the line BD and the tool tip motion trajectory. If the cutting edge reaches the point E at the time $t_{\rm E}$. The equation descibing the line BD in $O_1X_1Z_1$ can be written as:

 $z = k_{\rm BD}(x - x_{\rm B}) + z_{\rm B},\tag{11}$

where $k_{BD} = -\cot\beta$. According to the coordinate of point E obtained in Eq. (4), Eq. (11) can be expressed as:

$$\cot\beta (Rw_{\rm r}t_3 + A\sin(w_{\rm f}t_{\rm E} + \varphi_1) - x_{\rm B}) + z_{\rm B} - B\sin(w_{\rm f}t_{\rm E} + \varphi_2) = 0.$$
(12)

Then the separation time t_s can be expressed as a time difference when the cutting edge was at those respective locations:

266
$$t_{\rm s} = t_{\rm E} - t_{\rm B}.$$
 (13)

267 Now, the tool workpiece contact rate r can be given by:

268
$$r = 1 - \frac{t_s}{T} = 1 - \frac{t_E - t_B}{T}.$$
 (14)

According to Eqs (4), (10) and (14), it can be concluded that the tool workpiece contact 269 270 rate is affected by the ultrasonic frequency f, the ultrasonic amplitude A and B, the tool helix angle β , and spindle speed *n*. Fig. 4 shows the effects of different parameters 271 272 on the tool workpiece contact rate. In Fig. 4(a), three different amplitudes of 2,4 and 6μ m are set and as a result, the contact rate r is increased from 0.340 to 0.7659 at the 273 274 spindle speed n = 1200.53 rpm. It can be observed that the spindle speed is correlated with the contact rate r. As the spindle speed is increased, the ultrasonic vibration effect 275 276 is weakened and the cutting speed perpendicular to cutting edge direction increases. Thence the cutting time and contact rate increase. As illustrated in Fig. 4 (b), the tool 277 278 helix angle affects the decomposition of the velocity vectors. It can be concluded that the cutting time for a large tool helix angle in the LTUAM is lower than that for a small 279 280 tool helix angle.

Figs 4(c) and (d) plot the effects of ultrasonic parameters on the tool workpiece contact 281 282 rate. It can be seen that as the ultrasonic amplitude and frequency are increased, tool workpiece contact rate r drops up quickly. The main reason is consistent with the 283 variation of spindle speed. Besides, under the spindle speed n = 2000 rpm, the tool 284 285 workpiece contact rate for 20.25kHz and 39.75 kHz are found to be about 0.4655 and 0.3064, respectively. The net cutting time is reduced about 51.9% in each ultrasonic 286 287 cycle with the increased frequency from 20.25kHz to 39.75kHz. This means that the tool experiences a short duration of the pulsating cutting force when applying high 288 ultrasonic frequency as explained in [24] and [28]. Fig. 4(d) shows the three-289 290 dimensional surface diagram of vibration amplitudes on the contact rate r in LTUAM 291 process. It can be noted that when vibration amplitudes are small, the contact rate r292 maintains value 1. As vibration amplitudes are increased, the contact rate r becomes 293 small. Therefore, vibration amplitudes have a great influence on the contact rate r.



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Figure 4. Effects of different parameters on the tool workpiece contact rate r;(a) spindle speed($f = 20 \text{ kHz}, A = B = 6 \mu \text{m}$), (b) tool helix angle ($n = 2000 \text{ rpm}, A = B = 4 \mu \text{m}$); (c) ultrasonic frequency ($n = 1000 \text{ rpm}, A = B = 6 \mu \text{m}$); (d) ultrasonic amplitudes (f = 20 kHz, n = 1500 rpm).

299 **3. Micro dimpled surface generation model**

In this section, the relative motion between the tool and the workpiece and the mathematical description of the tool cutting edge are studied systematically. Finally, the ultrasonic surface topography prediction model is established considering the tool angles, which is also applicable in CM.

304 **3.1. Coordinate transformation**

The tool moves from $X_w Y_w Z_w$ to $X_f Y_f Z_f$ at a fixed feed velocity. Thence the translational vector can be calculated as:

$$\boldsymbol{F}_{o_{\mathrm{f}}}^{\mathrm{w}} = \begin{bmatrix} \boldsymbol{v}_{\mathrm{f}} \boldsymbol{t} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}.$$
(15)

The tool coordinate system $(X_tY_tZ_t)$ rotates with Z_f axis of the coordinate system $X_fY_fZ_f$. Therefore the rotational transformation matrix from $X_fY_fZ_f$ to $X_tY_tZ_t$ can be expressed as:

311
$$\boldsymbol{R}_{f}^{t} = \begin{bmatrix} \sin\theta & -\cos\theta & 0\\ \cos\theta & \sin\theta & 0\\ 0 & 0 & 1 \end{bmatrix}.$$
 (16)

According to Eqs (15) and (16), the transformation matrix from $X_w Y_w Z_w$ to $X_t Y_t Z_t$ can be written as:

314
$$\boldsymbol{R}_{w}^{t} = \begin{bmatrix} \boldsymbol{R}_{f}^{t} & \boldsymbol{F}_{o}^{w} \\ \boldsymbol{0} & 1 \end{bmatrix}.$$
 (17)

In the tool coordinate system, the discrete coordinate points of the tool cutting edge can be described as follows:

317
$$\boldsymbol{P}^{\mathsf{t}} = \begin{bmatrix} \boldsymbol{x}_{\mathsf{m},\mathsf{n}} \\ \boldsymbol{y}_{\mathsf{m},\mathsf{n}} \\ \boldsymbol{z}_{\mathsf{m},\mathsf{n}} \end{bmatrix}, \tag{18}$$

where all details with regards to the coordinate points $[x_{m,n}, y_{m,n}, z_{m,n}]$ will be explained in Section 3.2. Therefore, the discrete points of the tool cutting edge in $X_w Y_w Z_w$ can be given as:

$$\boldsymbol{P}^{\mathrm{w}} = \boldsymbol{R}_{\mathrm{w}}^{\mathrm{t}} \begin{bmatrix} \boldsymbol{P}^{\mathrm{t}} \\ 1 \end{bmatrix}.$$
(19)

After the motion between the tool and the workpiece is established, the tool and the workpiece should be discretized. Some detailed discussions can be found in Section 3.2.

324 **3.2. Description of tool geometry**

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The tool clearance angle is a key factor in the process of forming surface texture. Some 325 studies simplified the tool cutting edge as a straight line and arc as explained in [43]. 326 327 However, this is insufficient for LTUAM due to the phenomenon of recutting effect e.g. see [35]. Considering the longitudinal vibration of the tool, the 3D topography of the 328 329 tool should be discretized. In the process, the secondary cutting edge and the clearance angle of the tool are the primary contact parts between the tool and the workpiece, 330 331 therefore they are considered in the simulated model. Besides, the heights of the discrete points are different due to the structure of the milling cutter. In the tool radial direction, 332 the cutting edge is also discretized into discrete points. It can be seen from Fig. 5(a) that 333 the secondary cutting edge of the tool is simplified into a straight line and a circular arc. 334 335 In each circumferential direction, the tool is expanded in the circumferential direction 336 in Fig. 5(b). The clearance angle makes the discrete points of the tool have different 337 height values. The arc length $l_{m,n}$ that the tool is expanded in the circumferential 338 direction can be expressed as:

$$l_{m,n} = \theta_{m,n} r_{m,n}, \tag{20}$$

340 where $\theta_{m,n}$ and $r_{m,n}$ are the radian and radius at position (m,n). m and n are the

- number of discrete points in the tool radial direction and the circumferential direction, respectively. $m=1, 2, ..., M_1; n=1, 2, ..., N_1$.
- 343 As shown in Fig. 5(c), the cutting edge of the tool is discretized as grid points as follows:

344
$$\begin{cases} x_{m,n} = r_{m,n} \cos\theta_{m,n}, \\ y_{m,n} = r_{m,n} \sin\theta_{m,n}. \end{cases}$$
(21)

Considering the influence of ultrasonic vibration and the tool clearance angle on the height value in $X_t Y_t Z_t$, the value of secondary cutting edge $z_{m,n}$ can be written as:

347
$$\begin{cases} l_{m,n} \tan \alpha - r_{m,n} \tan \kappa + x_1 \tan \kappa + r_e (1 - \cos k), 0 < r_{m,n} \le x_1, \\ l_{m,n} \tan \alpha + r_e, x_1 < r_{m,n} \le R, \end{cases}$$
(22)

where $x_1 = R - r_e - r_e \sin \kappa$. α denotes as the clearance angle of the tool (rad), κ represents the angle of the secondary cutting edge of the tool (rad).

As shown in Fig. 5(d), the workpiece is discretized into grid points and the original height of the workpiece is stored in each grid point. In Fig. 5(e), for the points in the cutting area, if the height of the cutting edge is lower than the height of the workpiece, the workpiece height value will be updated. Otherwise, the workpiece height value will be kept unchanged. The specific update strategy can refer to the studies of Buj-Corral *et al.* [57]. Therefore, the ultrasonic surface topography can be obtained through Eq. (23) as shown below,

357
$$\begin{bmatrix} x_{m,n} \\ y_{m,n} \\ z_{m,n} \\ 1 \end{bmatrix}_{w} = \mathbf{R}_{w}^{t} \begin{bmatrix} x_{m,n} \\ y_{m,n} \\ z_{m,n} \\ 1 \end{bmatrix}_{t} = \begin{bmatrix} v_{f}t + r_{m,n}\sin(w_{r}t + A\sin(w_{f}t + \varphi_{1})/R + l_{m,n}/R) \\ r_{m,n}\cos(w_{r}t + A\sin(w_{f}t + \varphi_{1})/R + l_{m,n}/R) \\ z_{m,n} + B\sin(w_{f}t + \varphi_{2}) \\ 1 \end{bmatrix}$$
(23)

where \mathbf{R}_{w}^{t} denotes as the transformation matrix from $X_{w}Y_{w}Z_{w}$ to $X_{t}Y_{t}Z_{t}$ and ($r_{m,n}, z_{m,n}$) can be obtained from Eqs (21) and (22).



Figure 5. Discretization of the tool and the workpiece geometry; (a) clearance angle of the tool; (b) angle of the secondary cutting edge of the tool; (c) determination of the tool point cloud; (d) establishment of the workpiece point cloud; (e) update strategy of the workpiece point cloud.

360

364 3.3 Analysis of the micro dimpled surface generation process in 365 LTUAM

The micro dimpled surfaces are affected by machining and vibration parameters. And it is essential to understand the textured surface generation process so that the textured surface can be designed quantitatively. In this section, generation process of micro dimpled structures is mainly analyzed from the geometric aspect including the size, shape and geometric position under different parameters. Guo *et al.* [17] firstly defined the parameter λ_1 during the elliptical vibration turning. This ratio, λ_1 is defined as the vibration frequency over the spindle rotation speed as

$$\lambda_1 = 60f/n = k_1 + \varepsilon_1. \tag{24}$$

where k_1 and ε_1 are the integer part and fractional part of λ_1 . However, turning is different from milling owing to its different kinematics tool designs. In this paper, an improved definition of parameter λ_2 is given with respect to number of the tool teeth. The parameter λ_2 is the ratio of the ultrasonic frequency to the tool tooth frequency. Besides, λ_2 can also be divided into an integer part k_2 and a fractional part ε_2 , which can be expressed as:

380

 $\lambda_2 = 60f/nN = k_2 + \varepsilon_2,$

(25)

381 where *N* is the number of the tool teeth.

As shown in Fig. 6(a), in the feed direction the distance of the micro dimples formed by two adjacent teeth is equal to the feed per tooth. In the cutting direction, the distance d between two adjacent dimples along the cutting velocity can be determined as:

$$d = \frac{v_{\rm c}}{f} = \frac{\pi R n}{30 f}.$$
 (26)

According to Eq. (26), the larger distance d corresponds to higher spindle speed, which also results in a decrease in the density of the micro structures. Besides, the influences of ultrasonic frequency and the spindle speed on the value of d are opposite. In actual processing, the change of ultrasonic frequency is challenging due to the limitation of the resonance frequency of the ultrasonic structure. It is still essential to understand the effect of the ultrasonic frequency on the surface topography in LTUAM.

The phase shift distance of the adjacent dimples between the single tooth can be calculated from:

$$\delta_1 = \frac{\pi R n \varepsilon_1}{30 f}.$$
 (27)

While the phase shift distance of the adjacent dimples between the adjacent tooth can be determined as:

 $\delta_2 = \frac{\pi R n \varepsilon_2}{30 f}.$ (28)

As shown in Fig. 6(b), the maximum opening angle η between the tool path and the horizontal plane can be expressed as:

400
$$\eta = \operatorname{atan}\left(\frac{2A}{x_{\mathrm{F}} - x_{\mathrm{M}}}\right) = \operatorname{atan}\left(\frac{120Af}{\pi Rn - 120Af}\right). \tag{29}$$

401 where $x_{\rm F}$ and $x_{\rm M}$ are the abscissas of point F and M in $O_1 X_1 Z_1$. As shown in Figs 402 6(c)-(d), when $\alpha < \eta$, the tool clearance angle will intersect with the tool tip kinematic 403 trajectories at point P. Nevertheless, when $\alpha > \eta$, there will be no intersection between 404 the tool clearance angle and the tool tip kinematic trajectories.



405

421

406 Figure 6. Quantitative analysis of the micro dimpled surfaces; (a) schematic of the micro dimples 407 in LTUAM; (b) illustration of tool path; (c) and (d) cross-section profiles along the cutting direction 408 for (c) $\alpha < \varphi$ and (d) $\alpha > \varphi$.

409 According to Fig. 6(c), when the recutting effect is found, the distance between two 410 adjacent dimples consists of two parts, d_1 and d_2 . Point F is the lowest point of the 411 dimples. The coordinates of point F and point P can be obtained by Eqs (30), (31) and 412 (32) as follows:

413
$$y_{\rm P} = (x_{\rm P} - x_{\rm F})\tan\alpha + y_{\rm F},$$
 (30)

414
$$x_{\rm P} = v_{\rm c} t_{\rm P} + A \sin(w_{\rm f} t_{\rm P} + \varphi_{\rm 1}),$$
 (31)

415
$$z_{\rm P} = B\sin(w_{\rm f}t_{\rm P} + \varphi_2),$$
 (32)

416 where value of t_P is determined by the position of point F. And when point F is 417 determined, point P is also determined accordingly. So d_1 and d_2 can be expressed 418 as:

419
$$\begin{cases} d_1 = x_F - x_P, \\ d_2 = d - d_1. \end{cases}$$
 (33)

420 Finally, the value of h can also be calculated,

 $h = z_{\rm P} - z_{\rm F}.\tag{34}$

422 3.4 Effects of process parameters and tool angle on ultrasonic surface 423 topography

In this section, the effects of the process parameters and tool angle on the micro dimples are simulated. Specific parameters used in the simulation are shown in Table 1. The

426	suitable size of mesh grid should consider the selection of machining and ultrasonic
427	vibration parameters. In the simulation, ultrasonic amplitudes are the smallest physical
428	value and they are set to $2 \ \mu m$. In order to ensure the accuracy of the simulation, the
429	size of the workpiece mesh grid is set to 0.5µm. In order to study the relationship of
430	different parameters on the position and arrangement of the micro dimpled textures,
431	some quantitative results are calculated based on the development presented in Section
432	3.3. As the result different values of the parameter λ_1 and λ_2 are obtained from
433	simulations.

 434
 Table 1. Simulated parameters of surface textures for modelling LTUAM.

Parameters	
Spindle speed $n(rpm)$	1000,2000,3000,4000
Feed per tooth $f_z(mm/Z)$	0.01,0.02,0.03
Amplitude $A = B(\mu m)$	2
The tool clearance angle $\alpha(^{\circ})$	0°,5°,10°
The angle of secondary the cutting edge $\kappa(^{\circ})$	0°,2.5°,5°
The blunt radius of the tool tip $r_e(\mu m)$	2

435 Fig. 7 shows ultrasonic surface topography simulations for LTUAM under low spindle speed of 1000 and 2000rpm. It can be observed that as the feed per tooth increases, the 436 shape of the dimple becomes more wider in the feed direction. And the residual height 437 of the machining workpiece surface increases, which is consistent with the variation in 438 CM. In Figs 7(a)-(c), the arrangement of the micro dimpled texture is identical in the 439 440 cutting direction. And micro dimples are overlapping and continuous in the feed direction. Furthermore, the values of both the parameter λ_1 and λ_2 are 0 in Figs 7(a)-441 (c), which is the main reason that the same arrangement of the micro structures is 442 443 fabricated in the feed direction. In contrast, In Figs 7(d)-(f), the micro dimples are 444 intermittent in the feed direction and staggered in the cutting direction. And the phase 445 difference of the micro dimpled textures produced by the adjacent tool tooth is obviously found. At this time, the values of the parameter λ_1 and λ_2 are 0, 0.5, 446 respectively. The values of simulated error are calculated according to Section 3.3 and 447448 listed in Table 2. In the simulation, the tool clearance angle is 10° and the length of the 449 workpiece grid is 0.5 μ m. It can be found that the maximum error value of d is 1.6%. When compared with the previously reported models, the predicted surface topography 450 generated by the method proposed in this paper is closer to the actual surface 451 452 topography.



453

Figure 7. Micro dimpled texture simulations under the frequency f = 21.3kHz and different spindle speed and feed per tooth in LTUAM. The feed per tooth of (a)-(c) are 0.01 mm/Z, 0.02 mm/Z, 0.03 mm/Z, respectively, for the spindle speed n = 1000 rpm; the feed per tooth in (d)-(f) varies from 0.01 mm/Z, 0.02 mm/Z, 0.03 mm/Z, respectively, under the spindle speed n = 2000 rpm.

	(a)	(b)	(c)	(d)	(e)	(f)	(a)	(b)	(c)	(d)	(e)	(f)
			d(µ	lm)		$\delta_1(\mu m)$						
Theoretical value	14.74			29.49			0			0		
Simulated value	14.5		29.50			0			0			
The error	1.6%		0.03%			0			0			
	$f_{z}(\mu m)$						$\delta_2(\mu m)$					
Theoretical value	10 20 30		10	20 30		0		14.75				
Simulated value	10.0 20.0 30.0		10.0 20.0 30.0		0		14.5					
The error	The error $0 0 0$		0	0	0		0			1.7%		

458 Table 2 Values of the simulated errors for the surface topographies presented in Figure 7.

459

Fig. 8 depicts textured simulations for LTUAM under high spindle speeds of 3000 and 460 4000rpm. It can be found that dimensions of the micro dimples become large with the 461 462 improved spindle speed in the cutting direction. Besides, when the spindle speed is increased, a longer distance can be observed in the cutting direction, which can be 463 464 explained by the Eq. (26). According to Eqs (24) and (25), the values of k_1 and k_2 are decreased from 1278, 639 to 319, 159 with increase of the spindle speed, 465 respectively. This will decrease the density of the micro dimpled textures. Furthermore, 466 Figs 8(a)-(c) show the same discipline of the micro dimples with Figs 7(a)-(c). In Figs 467 8(d)-(f), the micro dimpled surface textures are arrayed with a greater deviation angle 468 in feed direction. This phenomenon can also be explained by the values of λ_1 and λ_2 . 469 The values of λ_1 and λ_2 in Figs 8(d)-(f) are 0.5,0.75, respectively. According to Eqs 470 471 (27) and (28), the phase shift distances in cutting direction are 29.49µm, 14.75µm, 472 respectively. Hence in Fig. 8(d), when feed per tooth is small, the overlapping areas between the adjacent dimples are more obvious in the cutting and feed directions. However, when compared with the condition of n = 3000 rpm, there is no significant increase in peak and valley values of h under n = 4000 rpm. This is a beneficial effect to generate a better surface roughness. And the overlapping areas between adjacent dimples in the feed direction are reduced with the increase of feed per tooth.

The values of the simulated error for Fig. 8 are listed in Table 3 according to the modelling introduced in Section 3.3. The error values of δ_1 , δ_2 and f_z in Figs 8(a)-(c) are both zero and the maximum error value of *d* is 5.6%. It can be noted that most of the errors are relatively small. Therefore, the simulation model is reliable and has a high accuracy. It can be concluded from Figs 7 and 8 that different surface structures can be obtained through the changing of machining and vibration parameters. And the values of λ_1 and λ_2 have a great impact on formation of the micro dimpled textures.



485

Figure 8. Micro dimpled texture simulation at the frequency f = 21.3kHz and different spindle speeds and feeds per tooth. The feed per tooth of (a)-(c) are 0.01mm/Z, 0.02mm/Z, 0.03mm/Z, respectively, under the spindle speed n = 3000rpm; The feed per tooth in (d)-(f) vary from 0.01mm/Z, 0.02mm/Z, 0.03mm/Z, respectively, with the spindle speed n = 4000rpm.

490 Table 3 Values of the simulated errors for the surface topographies presented in Figure 8.

	(a)	(b)	(c)	(d)	(e)	(f)	(a)	(b)	(c)	(d)	(e)	(f)	
	<i>d</i> (µm)							$\delta_1(\mu m)$					
Theoretical value	44.25			58.99			0			14.75			
Simulated value	ed value 44.0			59.0			0			15.0			
The error	5.6%		0.02%			0			1.69%				
	$f_{z}(\mu m)$								$\delta_2($	μm)			
Theoretical value	10 20 30		10	10 20 30		0		29.49					
Simulated value	10.0 20.0 30.0		10.0 20.0 30.0		0		30.0						
The error	0 0 0		0	0	0		0		1.73%				



493 Figure 9. Micro dimpled topography generated by adjacent teeth at the frequency f =494 20kHz and spindle speed n = 2000rpm and various phase differences of (a) 0, (b) 0.25π , (c) 495 0.5π , (d) 0.75π , (e) π , (f) 1.25π , (g) 1.5π , (h) 1.75π and (i) 2π .

492

496 The influence of the phase difference of the adjacent teeth on the ultrasonic surface topography is shown in Fig. 9, where the value of φ_1 is a constant, 0 and the value of 497 498 φ_2 is varying between 0 and 2π . It can be seen that when the value of φ_2 changes 499 between 0 and 2π , the micro dimples decrease firstly and then increase. And the 500 smallest micro dimples can be obtained when the value of φ_2 is equal to 0.5π . In contrast, when the value of φ_2 varies between π and 2π , the micro dimples increase 501 firstly and then decrease. And the biggest micro dimples can be obtained when the value 502 of φ_2 is set to $3\pi/2$. In this paper, the longitudinal and torsional vibration are coupled 503 together due to the structural design of the ultrasonic horn. The simulated analysis of 504 505 the influence of the phase difference can be used to determine the phase difference 506 accurately and guide the structural design of ultrasonic vibration equipment as studied in [58]. 507

- In addition, tool tip motion trajectories under different phase difference are analyzed in
- 509 Fig. 10. It can be found that if the sum of phase difference satisfies $\varphi_1 + \varphi_2 = 2\pi$, the
- ⁵¹⁰ ultrasonic surface topography is the same due to periodic movement of the cutting edge.

However, the tool tip motion trajectories alone are not sufficient to reflect the formation of the ultrasonic surface topography. According to Fig. 9, the generation process of the machined surface is the combined results between the tool cutting edge and the workpiece. The 3D tool topography has a significant effect on the final machined topography. Therefore, a detailed discussion how the tool kinematics affects the surface topography is given below.



517

518 Figure 10. Tool tip trajectories at the frequency f = 20kHz, spindle speed n = 2000rpm and 519 various phase differences of (a) 0, (b) 0.25π , (c) 0.5π , (d) 0.75π , (e) π , (f) 1.25π , (g) 1.5π , (h) 1.75π 520 and (i) 2π .

To investigate the effect of the tool angles on the ultrasonic surface topography, 521 522 different micro dimpled surfaces are obtained for different α and κ as shown in Fig. 11. It can be seen from Fig. 11(a) that when the value of κ is zero, the micro dimpled 523 524 surfaces cannot be fabricated. This is because the workpiece materials are cut by the 525 tool clearance angle in the cutting direction. However, as shown in Fig. 11(b), when the value of κ increased from zero to 2.5°, the micro dimples appear. Similarly, when the 526 value of α is zero, recutting effect occurs and micro dimpled surfaces are not 527 generated. As the value of α increases to 5°, micro dimples start to appear. 528 Consequently it can be stated that bigger values of α and κ result in more obvious 529 micro dimpled surfaces. 530



532 without considering the tool clearance angle. It can be seen that the simulated surface topographies are different owing to the influence of the tool clearance angle. As shown 533 534 in Fig. 11(f), the cross-section profiles obtained in Figs 11(d) and (e) are different from each other. And the value of η can be obtained to be 13.4° according to Eq. (29). Hence 535 when the tool clearance angle is considered, the cutting edge of the tool interferes with 536 the surface of the workpiece, which will further remove material. In general, the 537 proposed simulation model is reliable and in order to obtain a good typography of micro 538 539 dimpled surfaces, the values of α and κ must be set carefully.



540

Figure 11. Influence of tool angle on the ultrasonic surface topography for frequency f = 21.3kHz and spindle speed n = 2000rpm. The tool clearance angle α and secondary cutting edge angle κ are varied; (a) $\alpha = 10^{\circ}, \kappa = 0^{\circ}, (b)\alpha = 10^{\circ}, \kappa = 2.5^{\circ}, (c)\alpha = 0^{\circ}, \kappa = 5^{\circ}, (d) \alpha = 5^{\circ}, \kappa = 5^{\circ}$. (e) is obtained under the same parameters with (d) without considering the tool clearance angle. (f) depicts the cross-section profiles obtained in (d) and (e).

546 **4. Experimental setup**

Before the experimental studies were commenced, the ultrasonic vibration amplitude 547 calibration was conducted as shown in Fig. 12(a). Vibration signals were collected by 548 549 a laser displacement sensor with the sampling frequency is 80 kHz, where longitudinal and torsional responses were measured separately. The experimental results 550 demonstrated that the ultrasonic head can provide a stable amplitude in the range 551 between 0 and 5µm at frequency of 21.3 kHz, as shown in Figs 12(b) and (c). A high 552precision milling machine was used for all milling experiments of Ti-6Al-4V. The 553 554 diameter of the carbide milling cutter used in the experiment was 6 mm with the helix 555 angle of the cutter of 35°.



556

557 Figure 12. Amplitude calibration of the ultrasonic head; (a) vibration performance test setup; (b) 558 and (c) are the measured ultrasonic amplitudes of longitudinal and torsional vibration, respectively.



559

560 Figure 13. Experimental setup of LTUAM showing (a) experimental setup layout for LTUAM 561 experiments, (b) block schematic of LTUAM experiments, (c) surface topography testing machine 562 OLS4100, (d) hardness tester with measured samples.

The experimental setup of LTUAM is illustrated in Fig. 13(a) and it mainly consists of the ultrasonic milling machine and the signal acquisition system. As shown in Fig. 13(b), the LTUAM system includes an ultrasonic generator, a wireless transmission device and an ultrasonic vibration tool holder. The signal acquisition system is applied to collect the cutting force signal generated during the milling process. The sampling 568 frequency of the dynamometer is set to 7 kHz and matching charge amplifier and data acquisition system are used to record the data. Experimental parameters and tool 569 570 geometry for the end milling experiments are listed in Table 4. In the LTUAM process, the cutting force signals fluctuate within a certain range. The average cutting forces are 571 utilized to analyze the effect of different machining parameters on the cutting force. 572 After milling, the surface topographies are measured by a laser scanning confocal 573 microscope shown in Fig. 13(c). And surface micro hardness values in both CM and 574 575 LTUAM are measured by hardness tester, shown in Fig. 13(d), where the 500g load is applied to the indenter for 10s. 576

577 Table 4. Parameters and tool geometry for the end milling experiments.

Parameters						
Spindle speed <i>n</i> (rpm)	1000,2000,3000,4000					
Feed per tooth $f_z(mm/Z)$	0.01,0.02,0.03					
Amplitude $A, B(\mu m)$	1.5					
Frequency $f(kHz)$	21.3					
The tool clearance angle $\alpha(^{\circ})$	10°					
The angle of secondary the cutting edge $\kappa(^{\circ})$	5°					
The blunt radius of the tool tip $r_e(\mu m)$	2					

578 **5. Discussion and results**

In this section, the characteristics of material removal process and creating ultrasonically induced surface micro structures under different machining conditions will be analyzed. Modelling and experiments will be used to shed more lights on the nature of cutting forces, surface topographies and surface micro-hardness.

583 **5.1 Comparison of milling forces in CM and LTUAM**

The influences of different machining parameters in CM and LTUAM are investigated and shown in Fig. 14. Specifically, in Figs 14(a)-(c), both the milling force of CM and LTUAM are increased against the increased feed per tooth, and in LTUAM, the cutting force is significantly reduced. In order to eliminate the influence of the rotation position on the cutting force, the resultant force F_t and axial force F_z are used to analyze the experiment results. The resultant force F_t in the XOY plane can be expressed as:

590
$$F_{\rm t} = \sqrt{F_{\rm x}^2 + F_{\rm y}^2}.$$
 (35)





Figure 14. Comparison of cutting forces at different machining parameters; (a), (b) and (c) are the cutting force in x, y, z directions, respectively; (d) depicts the values of resultant force according to Eq. (33); (e) and (f) the reduction percentage of the resultant force F_t and axis force F_z .

595 Experimental data depicted in Figs 14(e) and (f) indicates that the tangential force and 596 forces in LTUAM are reduced between 3.11% and 28.90% and 12.70% and 45.05%

respectively, when compared with CM. The reduction reason can be explained by the 597 material removal mechanism in each ultrasonic cycle of LTUAM. As discussed in 598 Section 2.2, the cutting forces are different in CM and LTUAM. The material removal 599 of the tool cutting edge with ultrasonic vibration can change from continuous to 600 discontinuous cutting as the tool and workpiece are separated in each ultrasonic 601 vibration cycle. Besides, as is seen from Fig. 14(e), a more pronounced reduction of 602 milling force can be observed at lower spindle speed. This can be attributed to the 603 604 obvious separation effect, which facilitates a chip flowing and reduces a heat accumulation. As reported in [24], the amplitude of the cutting force can be also 605 significantly reduced. In ultrasonic assisted milling processing, although the cutting 606 thickness has a slight increase under the action of ultrasonic vibration as investigated 607 in [59]. The formation of chips is significantly improved and the friction between the 608 609 tool and the workpiece is reduced as shown in [60]. This has resulted in is a significant 610 reduction of milling forces.

611 **5.2 Verifying model of surface topography in LTUAM**

Comparisons of the 2D surface morphologies at n = 1000 rpm and n = 4000 rpm 612 613 in both CM and LTUAM are presented in Fig. 15. As shown in Figs 15(a)-(c), a large number of irregular feed trajectories and surface defects are clearly observed on the 614 machined surface in CM due to the plowing and rubbing. By contrast, the machined 615 616 surface appears substantial micro dimpled structures in LTUAM owing to the 617 application of ultrasonic vibration. It is worth noting that due to the smaller angle of the secondary cutting edge and the material properties of the titanium alloy, there are plastic 618 619 deformation of the machined material and metal plowing in Figs 15(j) and (k). And the 620 formation of the micro dimpled structure is improved as the feed per tooth is increased at n = 4000 rpm. In addition, the dimensions of the micro dimpled textures become 621 more larger with the improvement of the spindle speed. And the residual height of the 622 machined surface increases with the increased feed per tooth in both CM and LTUAM. 623 As analyzed in Section 2.1 and 2.2, the processing mechanism in LTUAM is completely 624 different to CM. The processing method, especially the tool tip trajectory, has a great 625 influence on the surface characteristics. In addition, some functional features such as 626 tribological performance and wettability were obtained by chosing relevant processing 627 and vibration parameters from the literature [61,62]. 628





629

Figure 15. Comparisons of the 2D surface morphologies in both CM and LTUAM. Specifically, the feed per tooth of (a)-(c) and (g)-(i) are 0.01mm/Z, 0.02mm/Z, 0.03mm/Z, respectively, for the spindle speedn = 1000rpm; the feed per tooth in (d)-(f) and (j)-(l) are varying from 0.01mm/Z, 0.02mm/Z, 0.03mm/Z, respectively, with the spindle speed n = 4000rpm.

634 635

055

637 In order to the verify the model of surface simulation in Section 3, some typical 638 morphologies of micro dimples are recorded under different parameters in Fig. 16. It can be observed that substantial micro dimpled textures are clearly found on the 639 640 ultrasonic machined surfaces. The number of microstructures is reduced and dimension becomes larger with the increased spindle speed and this can be explained by reduction 641 642 of k_1 and k_2 . The values of the parameter λ_1 and λ_2 are 0 in Figs 16(a) and (c), 643 thence the same micro dimples are fabricated in the feed direction and cutting direction. 644 The arrangement of the phase difference of the micro dimples in Figs 16(b) and (d) produced by the adjacent tool tooth is obvious, which indicates the simulated micro 645 dimpled surfaces are accurate. Furthermore, the surface profiles of experiment and 646 647 simulation in LTUAM are also compared in Figs 16(c), (f), (i) and (l). It can be noted that simulation and experimental results are matched well. In Fig. 16, there are some 648 649 irregular profiles which can be a side effect of the sampling strategies, which can influence 2D profiles. In the X direction, the same profiles for each feed are obtained 650 from the simulation. However, in the Y direction, due to reason that sampling areas are 651 small $(256 \times 256 \,\mu m^2)$ and the tool motion has trochoidal trajectories, irregular profiles 652 may be obtained. Surface textures in LTUAM have micro dimples and ridges, which 653 for perfectly matched process and ultrasonic parameters may result in unsmooth 654 655 profiles. Besides, the 3D tool shape can cause that tool paths of LTUAM to interfere 656 with the previous machining and this can also influence the 2D profiles. Experimental and simulation results of f_z and d are compared in Fig. 17. It can be found the 657 658 deviation of f_z and d vary from 1.95% to 6.31% and from 2.03% to 4.29%, respectively. It is worth noting that tool wear and incomplete material removal are the 659 660 primary reasons of theoretical and experimental deviation as explained in [11]. 661 However, the simulated surface topographies shown in Figs 7 and 8 still demonstrate the primary characteristics of the micro dimples. This indicates that the simulation 662 analysis is reliable. 663



Figure 16. Some typical morphologies obtained under different spindle speed and feed per tooth, specifically, the parameters of 2D, 3D morphologies and surface profiles in LTUAM are respectively (a), (b) and (c): $f_z = 0.02 \text{ mm/Z}$, n = 1000 rpm; (d), (e) and (f): $f_z = 0.02 \text{ mm/Z}$, n = 2000 rpm; (g), (h) and (i): $f_z = 0.04 \text{ mm/Z}$, n = 3000 rpm; (j), (k) and (l): $f_z = 670 \quad 0.06 \text{ mm/Z}$, n = 4000 rpm.



Figure 17. Comparisons between experimental and simulation results showing (a) the values of f_z and *d* obtained in experiment and simulation and (b) deviation analysis in (a).

5.3 Analysis of surface micro-hardness in CM and LTUAM

Surface micro-hardness is an important indicator for evaluating surface hardening and 675 the plastic deformation of the sub-surface layer is the main reason for work hardening, 676 677 which increases the strength and hardness of the surface material as reported in [63]. In addition, the improvement of surface micro hardness reduces the magnitude of 678 deformation induced by alternating loads, prevents fatigue-related crack growth and 679 improves fatigue performance. It can also help improve the wear resistance of 680 components. During milling, the surface and subsurface materials undergo severe 681 682 plastic deformation due to the strain hardening. Therefore, the microhardness of the machined surface is usually higher than that of the bulk material e.g. see [18]. 683



Figure 18. Influence of CM and RUEM on surface micro-hardness showing (a) micro-hardness value and (b) increased percentage between CM and LTUAM in (a).

684

Surface micro hardness values of both CM and LTUAM under different spindle speed 687 688 and processing methods are measured as illustrated in Fig. 18. It can be noted that surface micro hardness values in LTUAM are enhanced between 6.34% and 13.22% 689 compared with CM. Besides, lower spindle speeds and smaller feed speeds correspond 690 to higher micro-hardness values. Specifically, maximum surface hardness value 691 438.61HV can be obtained when the feed per tooth is 0.1 mm/Z and spindle speed is 692 1000rpm in LTUAM. This could be related with the deformation layer thickness of 693 the sub-surface layer. Zhang et al. [41] observed a thicker subsurface deformation of 694 Ti-4Al-6V at low cutting speed with rotary ultrasonic elliptical end milling. In addition, 695 the machined surface is further strengthened due to the effect of ultrasonic vibration, 696 which is similar to the mechanism of ultrasonic shot peening e.g. see [64]. It was 697 reported that finer grains and nanocrystalline layers in the subsurface layer were found 698

in ultrasonic machining as explained in [18,65], which could be primary reason ofincreased hardness in LTUAM.

701 **6. Conclusions**

In this work, machining characteristics including material removal kinematic process 702 and ultrasonic surface generation of LTUAM are modelled and experimentally verified. 703 The contact and separation behavior between the tool and workpiece in each ultrasonic 704 705 cycle are understood through an analytical model. In order to analyze the generation of 706 micro dimple textures, substantial numerical simulations are conducted using the 707 developed simulation method. The effects of machining and ultrasonic parameters on the micro dimpled structures are evaluated quantitatively and the main conclusions are 708 709 presented below.

The material removal process between tool and workpiece in LTUAM are studied analytically. The analysis of tool workpiece contact rate r demonstrates that the spindle speed is positively correlated with r, and ultrasonic frequency and the helix angle of the tool are negatively correlated with the r. Moreover, when vibration amplitudes are small, the contact rate r maintains 1 and as vibration amplitudes are increased, the contact rate r becomes lower.

- An analytical surface topography model of LTUAM is firstly established. Furthermore, the influence of 3D tool topography is investigated for the analysis of the formation of micro dimpled surfaces. Different types of surface topography are simulated and it was found that the clearance angle and the secondary cutting edge of the tool were essential to understand the generation of micro dimples structures.
- 721 The effects of process parameters including spindle speed, feed per tooth, ultrasonic 722 frequency) on the micro dimpled surface are discussed. The distance of micro dimpled 723 surfaces increases with the improvement of the spindle speed and the reduction of the 724 ultrasonic frequency. The position and arrangement of the micro dimpled textures can 725 be controlled by changing the values of the parameters λ_1 and λ_2 . Compared with 726 CM, the obvious reduction of the cutting force was observed in LTUAM. This can be attributed to the high frequency intermittent cutting mechanism due to the application 727 728 of ultrasonic vibration. It can be concluded ultrasonic vibration can change from continuous to discontinuous cutting. 729
- The micro hardness test results confirmed that LTUAM significantly enhanced the
 surface micro hardness from 6.34% to 13.22% and the improvement of the surface
 micro hardness are decreased as the spindle speed is improved. In addition, maximum
- surface micro-hardness value 438.61HV can be obtained in LTUAM at n = 1000rpm.

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